# Numerical Methods for the DNLS Equation

## Background and Galilean Transformation

1)  … the DNLS equation with dissipation and a driver

1’) 

A Galilean transformation to a frame of reference travelling with velocity, v, can be implemented through:

2)  and 

The effect of this transformation on the equation is found through constructing the exact derivative of a function such as . Given this and (2), then

3)  as well as 

Where (2) has been used to eliminate dx’. Equating multiples of dx and dt gives the transformed partial derivatives:

4)  and 

Substituting these transformed derivatives into (1’) and then dropping the primes yields:

1’’)  … the DNLS under a Galilean transformation to a reference frame moving at speed, v.

Next, Fourier transform the equation using

5)  … where B(k,t) is the Fourier Transform of b(x,t)

Resulting in,

6)  … with F[g(x)] being the transform of g(x)

## The Leapfrog Method

We will implement a finite difference method in time known as the Leapfrog Method. As is typical in finite difference methods, the continuous variable, t, is replaced with a discretized variable, tn:

7) tn = n\*t where t is the step size in time related to x by t = Courant\*x with n = 0, 1, 2, …

where x is defined by,

8) , where  and N is the number of points for which b(x,t = 0) is sampled.

With this understood, then the finite difference approximation for Bt is taken to be:

 where , so the superscript, n, is just the index referring to a time step.

The Leapfrog Method takes B(t) of the k2 term of (6) to be the average in time over  and  so, and the others as 

, solving for  yields:

9) 

In order to implement this method iteratively, one must be given both , the initial profile, as well as . The latter can be determined using the Split-Step Method.

## The Split-Step Method

This method takes  in two steps as:







